b. Prove that every compact subset A of a Hausdorff space X is compact. Give an example of a compact space which is not Hausdorff.

c. Prove that a one one continuous map of a compact space onto a Hausdorff space is homeomorphic

4. **a.** Prove that – Every second countable space is separable. 6+4 =10

b. State Urysohn lemma and the Tietze extension theorem.

5. a. Prove that – The union of a collection of connected subspaces of X that have a common point is connected. 5+5=10

b. Show that if X is an infinite set, it is connected in the finite complement topology.

6. **a.** Prove that - Every interval in $\mathbb{R}$ is connected. 6+4 =10

b. State and prove the Intermediate value theorem.

7. **a.** Define countable and uncountable sets.Prove that a countable union of countable sets is countable. 5+5=10

b. What is cardinal number. Let α, β *and* γ be cardinal numbers, then prove that

i. $\alpha \leq \alpha$

ii. $\alpha \leq \beta$

iii. $\beta \leq \gamma \Rightarrow \alpha \leq \gamma$

8. A relation R on a topological space $(X, \mathcal{T})$ is defined as follows: 5+5 =10

$R = \{(x, y) \in X \times X : x, y \in E_{xy}\}$

Where E_{xy} is a connected subset of X. Show that –

a. R is an equivalence relation.

b. Define equivalence class with respect to R and prove that equivalence class form a partition on X.

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M. Sc. MATHEMATICS
SECOND SEMESTER
TOPOLOGY
MSM - 201

(Use Separate Answer Scripts for Objective & Descriptive)

Duration : 3 hrs. **Full Marks : 70**

[PART-A : Objective]

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1×20=20***

1. The image of a connected space is connected if the function is:
   a. Derivable b. Continuous
   c. Continuous and derivable d. None of these
2. Consider the following two statement:
   P: An indiscrete topological space is a T_0-space.
   Q: A discrete topological space is a T_0-space.
   a. Both P and Q are true. b. P is true but Q is false.
   c. Q is true but P is false. d. Both P and Q are false.
3. Consider the following statement:
   P: If $A \subseteq \mathbb{R}$ is not an interval then A is connected.
   Q: If $A \subseteq \mathbb{R}$ is an interval then A is disconnected
   a. Both P and Q are true b. Neither P nor Q is true
   c. Only P is true d. Only Q is true.
4. Let X and Y be topological space and let $f: X \to Y$ be a continuous map. For any subset S of X, which one of the following is true?
   a. If S is open then $f(S)$ is open. b. If S is closed then $f(S)$ is closed
   c. If S is connected then $f(S)$ is connected d. If S is bounded then $f(S)$ is bounded
5. Consider the following statement:
   P: Every Lindelof space is second countable space.
   Q: Every second countable space is Lindelof.
   a. Both P and Q are true. b. P is true but Q is false.
   c. Q is true but P is false. d. P and Q are fasle.
6. Let $f: [a, b] \to [a, b]$ be a continuous function. Consider the following statements:
   P: there exists $x_0 \in [a, b]$ such that $f(x_0) = x_0$.
   Q: there exists $x_0 \in (a, b)$ such that $f(x_0) = x_0$.
   a. Only Q is true. b. Only P is true.
   c. Both P and Q are false. d. Both P and Q are true.
7. Let (X, T) be a topological space. Define $R = \{(x, y) \in X \times X: x, y \in E_{xy}\}$, wheer E_{xy} is connected subspace of $X$and $[x] = \{y \in X: yRx\}$. Then
   a. R is an equivalence relation. b. $[x]$is the maximal connected space containing x.
   c. Both are (a) and (b) are true. d. None of these is true.

P.T.O.

8. Let $X = \{a, b, c\}$.Which of the following is not a topology on X.
a. $\{\emptyset, X\}$ b. $\{\emptyset, \{a\}, X\}$ c. $\{\emptyset, \{a\}, \{b\}, \{c\}, X\}$ d. None of these

9. The derived set of $(0,1)$ is
a. $(0,1]$ b. $[0,1)$ c. $(0,1)$ d. $[0,1]$

10. The topologist's sine curve is
a. Connected but not locally connected
b. Locally connected but not connected
c. Connected as well as Locally connected
d. None of these

11. Let A be a set, then
a. There exists a bijection of A with a proper subset of itself
b. There exists a bijective function $f: Z_+ \to A$
c. There exists a surjective function $f: Z_+ \to A$
d. None of these

12. Which of the following statement is true for a subset A of a topological space.
a. $\overline{A}$ is the largest closed set conataing A
b. If A is closed, then A contains all its limit points
c. A is closed if and only if $\overline{A} \neq A$
d. None of these

13. Which of the following statement is not true
a. A subset of R is compact if and only if it is closed and bounded
b. Every subset A of a Hausdorff space is closed.
c. A metric space X is sequentially compact if and only if every finite subset of X has a limit point.
d. None of these

14. Let $(X, \mathcal{T})$ and $(Y, \mathcal{V})$ be two topological spaces. Then the topology $\mathcal{W}$ whose base is $E = \{G \times H : G \in \mathcal{T} \text{ and } H \in \mathcal{V}\}$ is called the
a. Quotient topology for X and Y
b. Product topology for X and Y
c. Metric topology for X and Y
d. None of these

15. For any cardinal number α,
a. $\alpha > 2^\alpha$ b. $\alpha \leq 2^\alpha$ c. $\alpha \geq 2^\alpha$ d. $\alpha < 2^\alpha$

16. If B is base for a topological space (X, T), then every T open set can be expressed as
a. Intersection of members of B
b. Union of members of B
c. Difference of members of B
d. None of these

17. Which of the following is not a neighbourhood of 1
a. (0,2) b. (0,2] c. [1,2] d. R

18. If d = cardinal number of the set of natural numbers and c =cardinal number of the set of rational numbers, then
a. $d = c$ b. $d > c$ c. $d < c$ d. $d \leq c$

19. If $X = \{a, b, c\}$ and $T = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$,then closure of {a} will be
a. {a} b. {a,b} c. {a,c} d. None of these

20. If f is a map from a topological space X to a topological space Y and $\{a\}$ is open in X, then f is
a. Continuous at a
b. Discontinuous at a
c. Cannot be said
d. None of these

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(PART-B : Descriptive)

Time : 2 hrs. 40 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. a. Give the definition of a base in a topological space. Let $X = \{a, b, c, d\}$ and $\mathcal{T} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}, \{b, c, d\}, \{c, d\}\}$.Then show that the collection $\mathcal{B} = \{\{a\}, \{b\}, \{c, d\}\}$ is a base for $\mathcal{T}$. 3+3+4 =10

b. If $(X, \mathcal{T})$ is a topological space and $\mathcal{B}$ is a base for $\mathcal{T}$, then prove that intersection of any two members of $\mathcal{B}$ is the union of members of $\mathcal{B}$.

c. Define Hausdorff space in a topological space. Examine whether the following spaces are Hausdorff or not.

(i) $X = \{a, b, c\}, \mathcal{T} = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$

(ii) $X = \{a, b, c, d\}$

$\mathcal{T} = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

2. a. Define continuity and homeomorphism in a topological space. 2+2+3+3 =10

b. If $(X, \mathcal{T})$ and $(Y, \mathcal{V})$ are two topological spaces such that

$X = \{a, b, c\}, \mathcal{T} = \{\emptyset, \{a\}, \{b, c\}, X\}$ and

$Y = \{p, q, r\}, \mathcal{V} = \{\emptyset, \{r\}, \{p, q\}, Y\}$.

A function $f: X \to Y$ is defined by $f(a) = r, f(b) = p, f(c) = q$. Show that f is continuous at each points of X. Also show that f is homeomorphism.

c. Let $(X, \mathcal{T})$ and $(Y, \mathcal{V})$ be two topological spaces. Prove that a mapping $f: X \to Y$ is continuous if the inverse image under f of every closed set in Y is closed in X.

d. Let $(X, \mathcal{T})$ and $(Y, \mathcal{V})$ be two topological spaces and $f: X \to Y$ be bijective mapping. Prove that f is homeomorphism if f is continuous and open.

3. a. Define compactness in a topological space. Show by an example that a topological space X is compact if X is finite. 4+6 =10