c) Classify the following integral equation as Fredholm or Volterra integral equation, Linear or Non-linear and Homogenous or Non-homogeneous. Justify your answer.

(i) $u(x) = \int_0^x (x - t) u(t) dt$

(ii) $u(x) = \frac{2}{7}x + \int_0^1 xt \ u^2(t) \, dt$

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REV-00 2018/12
MSP/58/63

M.Sc. PHYSICS
THIRD SEMESTER
MATHEMATICAL PHYSICS-II
MSP-301

(Use separate answer scripts for Objective & Descriptive)

Duration: 3 hrs. **Full Marks: 70**

(PART-A : Objective)

Time: 20 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. The orthogonality condition of Hermite polynomial for (m $\neq n$) is given by:

a. $\int_{-\infty}^{+\infty} H_m(x) H_n(x) \, e^{-x^2} dx = 0$ **b.** $\int_{-\infty}^{+\infty} \varphi_m(x) \varphi_n(x) dx = 0$

c. $\int_{-\infty}^{+\infty} \varphi_m(x) \varphi_n(x) e^{\ x^2} dx = 0$ **d.** $\int_{-\infty}^{+\infty} H_m(x) H_n(x) \, e^{x^2} dx = 0$

2. If a group possesses an element such that $I, a, a^2, a^3, \ldots$ includes............ of a group, it is called a cyclic group.

a. All elements **b.** Two elements

c. Few elements **d.** None of these

3. The number of symmetry operations of an equilateral triangle that forms a finite group is:

a. Ten **b.** Six

c. Four **d.** None of these

4. The orthogonal properties of Laguerre polynomial L_n is given by

a. $\int_0^{\infty} e^{x} L_n(x) L_m(x) dx = 0$ **b.** $\int_0^{\infty} L_n(x) L_m(x) dx = 0$

c. $\int_0^{\infty} e^{-x} L_n(x) L_m(x) dx = 0$ **d.** None of these

5. Choose the incorrect option from the following:

a. A cyclic group is also abelian.

b. An abelian group is also cyclic.

c. If '*a*' group possesses an element $a, a^2, a^3, \ldots$, then the group is cyclic.

d. All of the above

6. If a subset G' and G is closed under.............. it is also a group and called a sub-group.

a. Rotation **b.** Addition

c. Multiplication **d.** None of these

7. The recurrence formula for Bessel function of the form $x^{-n} J_n(x)$ is equal to:

a. $x^n J_n$ **b.** $-x^{-n} J_{n+1}$

c. $x^n J_{n+2}$ **d.** $x^n J_{n-1}$

8. The equation $g(x) = f(x) + \int_a^b dt \, K(x, t) f(t)$ is a:

a. Volterra equation of first kind **b.** Volterra equation of second kind

c. Fredholm equation of first kind **d.** Fredholm equation of second kind

9. If the Kernel $K'(x,t) = K(t,x)$, it is:
   a. Asymmetric
   b. Continuous
   c. Discontinuous
   d. Symmetric

10. The matrix of linear transformation T: R →R^2 defined by T(x)= (6x, 8x) with respect to a standard basis is:
   a. $[6 \quad 8]$
   b. $[8 \quad 6]$
   c. $\begin{bmatrix} 6 \\ 8 \end{bmatrix}$
   d. $\begin{bmatrix} 8 \\ 6 \end{bmatrix}$

11. The eight SU (3) generators can be represented in terms of zero-trace Hermitian matrices with $\varsigma_i = \frac{1}{2}\lambda_i$. The λ_i are known as:
   a. Diagonal matrices
   b. Unitary matrices
   c. Gell-Mann matrices
   d. All of the above

12. The zero operator is a:
   a. Identity operator
   b. Zero transformation
   c. Linear operator
   d. None of these

13. The standard basis of R^3 is (e_1,e_2,e_3). Here e_1 is:
   a. (0,1,0)
   b. (1,0,0)
   c. (0,0,1)
   d. (1,0,1)

14. Lie's essential idea was to establish a group in terms of its:
   a. Representation
   b. Generators
   c. Parameters
   d. None of the above

15. The operator $dx_1 |x_1\rangle\langle x_1|$ is called:
   a. An ordinary propagator
   b. A propagator
   c. An identity operator
   d. All of the above

16. The order of a vector field R^4 is:
   a. 4
   b. 2
   c. 3
   d. None of these

17. The number of generators of a Lie group is equal to the:
   a. Basis of the group
   b. Parameter of the group
   c. Order of the group
   d. None of the above

18. Irreducible representations of abelian group are:
   a. 2×2
   b. $n \times n$
   c. 1×1
   d. None of the above

19. The value of Hermite polynomial $H_0(x)$ is:
   a. 0
   b. 1
   c. ($4x^2$- 2)
   d. None of these

20. The solution of $P_n(x)$ and $Q_n(x)$ is a series of one of the following kind:
   a. both $P_n(x)$ and $Q_n(x)$ are terminating.
   b. $P_n(x)$ is non-terminating and $Q_n(x)$ terminating.
   c. both $P_n(x)$ and $Q_n(x)$ are non-terminating.
   d. $P_n(x)$ is terminating and $Q_n(x)$ non-terminating.

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(PART-B :Descriptive)

Time: 2 hrs. 40min. Marks: 50

[Answer question no.1 & any four (4) from the rest]

1. a) Prove the orthogonality of Bessel function. 5+5=10
   b) Prove that $\int x J_0^2(x)dx = \frac{1}{2}x^2\left[J_0^2(x) + J_1^2(x)\right] + C$

2. a) Express the following function in Fourier-Legendre expansion. 5+5=10
   $f(x) = \begin{bmatrix} 0 & -1 \le x \le 0 \\ x^2 & 0 \le x \le 1 \end{bmatrix}$
   b) Prove that: $(n+1)P_{n+1} = (2n-1)xP_n - nP_{n-1}$

3. a) Show that the mapping 4+6=10
   $f: V_3(R) = V_2(R)$ defined by $f(a,b,c) = (c, a+b)$ is a linear transformation
   b) Show that the mapping
   $f: V_3(R) = V_2(R)$ defined by $f(a,b,c) = (a-b, a+c)$ is a linear transformation

4. a) What is a unitary group? Show the Unitary representation of a group D_3 and hence show the reducible representation of an equilateral triangle. 4+3+3=10
   b) Write a brief note on Homomorphism and Isomorphism of a group.
   c) Verify whether there is Homomorphism or Isomorphism between groups of a Non-zero complex numbers (under multiplication) and complex numbers with absolute value 1(under multiplication).

5. a) Discuss about Lie group and their generators. 5+5=10
   b) Show that there is homomorphism between SU(2) and SO(3) generators.

6. State the theorems of Schur's Lemmas. Prove the Schur's second lemma. 4+6=10

7. a) Prove that: 2+2+6=10
   (i) $J_1''(x) = (n^2 - n - x^2)J_n(x) + xJ_{n+1}(x)$
   (ii) Prove that $P_n(1) = 1$
   b) Obtain the integral form of Linear harmonic oscillator equation by transformation of its differential form into homogenous Fredholm Integral equation of second kind.

8. a) If $u(x) = e^{-x^2}$ is a solution of the Volterra integral equation 3+3+4=10
   $u(x) = 1 - \alpha\int_0^x t\,u(t)\,dt$, Find α?
   b) Check if $u(x) = x + e^x$ is a solution of the Fredholm integral equation $u''(x) = e^x - \frac{4}{3}x + \int_0^1 xt\,u(t)\,dt;\ u(0) = 1,\ u'(0) = 2$