

35. (i)  $\int \frac{dx}{(1-x)\sqrt{x}}$

(ii)  $\int \frac{\sqrt{x} dx}{x-1}$  [ Put  $x = z^2$ . ]

36. (i)  $\int \frac{dx}{x\sqrt{(x^2 \pm a^2)}}$

(ii)  $\int \frac{dx}{(1+x)\sqrt{(1-x^2)}}$

(iii)  $\int \frac{dx}{x\sqrt{(9x^2 + 4x + 1)}}$

(iv)  $\int \frac{dx}{(1+x)\sqrt{(1+2x-x^2)}}$

(v)  $\int \frac{dx}{x\sqrt{(x^2 + 2x - 1)}}$

(vi)  $\int \frac{dx}{(1+x)\sqrt{(1+x-x^2)}}$

(vii)  $\int \frac{dx}{(x-3)\sqrt{(x^2 - 6x + 8)}}$

37. (i)  $\int \frac{\sqrt{(a^2 - x^2)}}{x} dx$

(ii)  $\int \frac{dx}{x + \sqrt{(x-1)}}$

38.  $\int \frac{dx}{x\sqrt{(1+x^3)}}$

[ Put  $1+x^3 = z^2$  ] [ C. P. '81 ]

39. (i)  $\int \sqrt{\frac{a+x}{x}} dx$

(ii)  $\int \sqrt{\frac{x-a}{x}} dx$

40. If  $a < x < b$ , show that

$$\int \frac{dx}{(x-a)\sqrt{\{(x-a)(b-x)\}}} = \frac{2}{a-b} \sqrt{\frac{b-x}{x-a}}$$

**ANSWERS**

1.  $\tan^{-1}(x^3)$ . 2. (i)  $\frac{1}{2} \tan^{-1}(x^2)$ . (ii)  $\frac{1}{4} \log \frac{x^2 - 1}{x^2 + 1}$ .

3. (i)  $\tan^{-1}(e^x)$ . (ii)  $\frac{1}{4} \sin^{-1}\left(\frac{x}{a}\right)^4$ . 4. (i)  $\tan x - \tan^{-1} x$ .

(ii)  $\frac{1}{4} \log \frac{2 - \cos x}{2 + \cos x}$ . 5. (i)  $\frac{1}{2} \log(x^2 + \sqrt{x^4 + a^4})$ .

METHOD OF SUBSTITUTION

$$23. \frac{1}{\sqrt{2}} \log \left( x + \frac{3}{4} + \sqrt{x^2 + \frac{3}{2}x + 2} \right).$$

$$24. 2 \log (\sqrt{x-3} + \sqrt{x-4}).$$

$$25. \sqrt{\frac{2}{5}} \sin^{-1} \sqrt{\frac{10x+4}{19}}.$$

$$26. -\frac{2}{\sqrt{5}} \log \{ \sqrt{2-5 \sin x} + \sqrt{5(2-\sin x)} \}.$$

$$27. 2 \log (\sqrt{x-\alpha} + \sqrt{x-\beta}).$$

$$28. (i) \sin^{-1} \left( \frac{x-a}{a} \right), \quad (ii) \log \left( x+a + \sqrt{x^2+2ax} \right).$$

$$29. (i) \sqrt{x^2+a^2} + b \log \left( x + \sqrt{x^2+a^2} \right).$$

$$(ii) 2\sqrt{x^2+x+1} + 2 \log \left( x + \frac{1}{2} + \sqrt{x^2+x+1} \right).$$

$$30. \frac{1}{2} \sqrt{2x^2-8x+5} \quad 31. (i) \frac{9}{5\sqrt{5}} \sin^{-1} \left( \frac{5x-4}{6} \right) - \frac{1}{5} \sqrt{4+8x-5x^2}.$$

$$(ii) \frac{1}{2} \sqrt{4x^2+4x+2} - \log \left( 2x+1 + \sqrt{4x^2+4x+2} \right).$$

$$32. (i) 2 \tan^{-1} (\sqrt{1+x}), \quad (ii) \frac{1}{2} \log \left( \frac{\sqrt{4x+3}-1}{\sqrt{(4x+3)+1}} \right).$$

$$33. \log \frac{\sqrt{2x+1}-\sqrt{3}}{\sqrt{(2x+1)+\sqrt{3}}}.$$

$$34. (i) \sqrt{(x-3)(x-4)} + \log (\sqrt{x-3} + \sqrt{x-4}).$$

$$(ii) \frac{1}{6} \left[ 2\sqrt{(2x+1)(3x+2)} - \frac{\sqrt{2}}{3} \log (\sqrt{3}\sqrt{2x+1} + \sqrt{2}\sqrt{3x+2}) \right].$$

$$35. (i) \log \frac{1+\sqrt{x}}{1-\sqrt{x}}.$$

$$(ii) 2\sqrt{x} + \log \frac{1-\sqrt{x}}{1+\sqrt{x}}.$$

$$36. (i) \frac{1}{2a} \log \frac{\sqrt{x^2+a^2}-a}{\sqrt{(x^2+a^2)+a}}; \quad \frac{1}{a} \sec^{-1} \frac{x}{a} \quad (ii) -\sqrt{\frac{1-x}{1+x}}.$$

$$(ii) \log \{(1+x^2 + \sqrt{1+x^4})/x\}.$$

$$6. \sin^{-1} \sqrt{\frac{x^2 - a^2}{b^2 - a^2}}.$$

$$7. (i) \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right).$$

$$(ii) \frac{1}{4} \tan^{-1} \left( x + \frac{1}{2} \right).$$

$$8. (i) \frac{1}{\sqrt{5}} \log \frac{\sqrt{5} + 2x - 1}{\sqrt{5} - 2x + 1}. \quad (ii) \log \frac{2x+1}{3x-2}.$$

$$9. \frac{1}{2} \tan^{-1}(x^2 + 1). \quad 10. \frac{1}{2} \log \frac{1 + \sin x}{3 + \sin x}. \quad 11. \frac{1}{2} \tan^{-1} \left\{ \frac{1}{2} (e^x + 1) \right\}.$$

$$12. \tan^{-1}(\sin^{-1} x). \quad 13. \frac{1}{12} \log \frac{x^3 - 5}{x^3 - 1}. \quad 14. \frac{1}{3} \log \frac{2 + \log x}{5 + \log x}.$$

$$15. (i) \log(x+1) + \frac{1}{x+1}. \quad (ii) -\log(x-3).$$

$$16. (i) \frac{1}{2} \log(x^2 + 4x + 5) - \tan^{-1}(x+2).$$

$$(ii) \frac{1}{4} \log(4x^2 + 1) + \frac{3}{2} \tan^{-1}(2x).$$

$$17. (i) \frac{2}{3} \log(3x^2 + 3x + 1) + \frac{2}{\sqrt{3}} \tan^{-1} \{ \sqrt{3}(2x+1) \}.$$

$$(ii) \frac{-3}{2\sqrt{11}} \log \frac{\sqrt{11} + 3 + x}{\sqrt{11} - 3 - x} - \frac{1}{2} \log(2 - 6x - x^2).$$

$$18. x + \log \frac{x-2}{x+2}.$$

$$19. (i) x - 2 \tan^{-1}(x+1).$$

$$(ii) x - \log(x^2 + x + 1) + \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right).$$

$$20. \frac{1}{2} x^2 + 2x + \frac{3}{2} \log(x^2 - x + 1) + \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{2x-1}{\sqrt{3}} \right).$$

$$21. 2 \log(\sqrt{x+2} + \sqrt{x-1}).$$

$$22. (i) \sin^{-1} \left( \frac{2x+1}{\sqrt{5}} \right). \quad (ii) \log \left( 2x+3 + 2\sqrt{3+3x+x^2} \right)$$

$$23. \frac{1}{\sqrt{2}} \log \left( x + \frac{3}{4} + \sqrt{x^2 + \frac{3}{2}x + 2} \right).$$

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$$36. (i) \frac{1}{2a} \log \frac{\sqrt{x^2+a^2}-a}{\sqrt{(x^2+a^2)+a}}; \frac{1}{a} \sec^{-1} \frac{x}{a}. \quad (ii) -\sqrt{\frac{1-x}{1+x}}.$$

$$(iii) \log x - \log \left( 1 + 2x + \sqrt{9x^2 + 4x + 1} \right). \quad (iv) \frac{1}{\sqrt{2}} \sin^{-1} \left( \frac{x\sqrt{2}}{1+x} \right).$$

$$(v) \sin^{-1} \left( \frac{x-1}{x\sqrt{2}} \right). \quad (vi) \sin^{-1} \left( \frac{3x+1}{(1+x)\sqrt{5}} \right).$$

$$(vii) \sin^{-1}(x-3). \quad 37. (i) \sqrt{a^2 - x^2} + a \log \frac{a - \sqrt{a^2 - x^2}}{x}.$$

$$(ii) \log(x + \sqrt{x-1}) - \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2\sqrt{x-1} + 1}{\sqrt{3}} \right).$$

$$38. \frac{2}{3} \log \left( \sqrt{1+x^3} - 1 \right) - \log x.$$

$$39. (i) a \log \left( \sqrt{x} + \sqrt{x+a} \right) + \sqrt{x(x+a)}.$$

$$(ii) 2\sqrt{x-a} - 2\sqrt{a} \tan^{-1} \left( \sqrt{\frac{x-a}{a}} \right).$$