2023

SET A

M.Sc. MATHEMATICS
FOURTH SEMESTER
DYNAMICAL SYSTEM
MSM – 404B [SPECIAL REPEAT]
[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks:70**

(Objective)

Time: 30 min. **Marks:20**

Choose the correct answer from the following: ***1X20=20***

1. The linear system of equation $\dot{x} = y$, $\dot{y} = -2x - 2y$ have
   a. Spiral source
   b. Spiral sink
   c. Saddle point
   d. None

2. If x^* is an equilibrium point for the equation $\dfrac{dx}{dt} = f(x)$ then, x^* is a sink if
   a. $f'(x^*) > 0$
   b. $f'(x^*) < 0$
   c. Both A and B
   d. None

3. If $P(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots\ldots\ldots a_{n-1}\lambda + a_n$

   Where $a_1, a_2, \ldots\ldots$ are real coefficients and then the Hurwitz matrix is
   a. $H_2 = \begin{bmatrix} a_1 & 1 \\ a_3 & a_2 \end{bmatrix}$
   b. $H_2 = \begin{bmatrix} a_1 & 1 \\ a_2 & a_2 \end{bmatrix}$
   c. Both A and B
   d. None

4. If $P(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots\ldots\ldots a_{n-1}\lambda + a_n$ then all the roots of the polynomial $P(\lambda)$ are negative or have negative real part iff the determinants of the all Hurwitz matrices are
   a. Positive
   b. Negative
   c. Zero
   d. None

If the coefficients of the above characteristic polynomial $P(\lambda)$ are positive then roots of the characteristic polynomial are

a. negative
b. Having negative real part
c. Both A and B
d. None

For a non linear system of dynamical system if the eigen values λ_1 and λ_2 are real, unequal and both are positive then the equilibrium point is

a. Unstable node
b Source
c. Repeller
D All of the above

The linear system of equation

$$\dot{x} = -4x + y$$
$$\dot{y} = 3x - 2y$$

have

a Stable node
B Unstable node
c Stable point
D None

For a non linear system of dynamical system if the eigen values λ_1 and λ_2 are complex having negative real part then the equilibrium point is

a. Spiral Sink
b. Stable spiral
c. Both A and B
d. None

For a linearised system of dynamical system if the eigen values λ_1 and λ_2 are real and equal then the system is unstable at origin if

a. $\lambda_1 = \lambda_2 < 0$
b. $\lambda_1 = \lambda_2 > 0$
c. Both A and B
d. None

In Lyapunov theorem if V(x,t) be the Lyapunov function then for global stability $\dot{V}(x,t)$ must be

a. Negative
b. Positive
c. Negative definite
d. Positive definite

If x^* is an equilibrium point for the equation $\frac{dx}{dt} = f(x)$ and if $f'(x^*) = 0$ then x^* is a /an

a. Semi stable point
b. Unstable
c. Needs further investigation
d. All of the above

12. The following linear system of equation

$$\dot{x} = -2x - 6y + 8$$
$$\dot{y} = 8x + 4y - 12$$

have

a. Spiral source
b. Spiral node
c. Unstable sink
d. None

13. If x^* is an equilibrium point for the equation $\frac{dx}{dt} = f(x)$ then, x^* is a sink if

a. $f'(x^*) > 0$
b. $f'(x^*) < 0$
c. Both A and B
d. None

14. For a matrix A the characteristic equation or polynomial is given by

a. $|A - \lambda I| = 0$
b. $A - \lambda I = 0$
c. $|A - \lambda I| \neq 0$
d. None

15. Sink is an equilibrium point up to which the solution curve of a system

a. Seems to converge as $t \to \infty$
b. Sufficiently close as $t \to \infty$
c. Asymptotic as $t \to \infty$
d. D

16. If $P(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots\ldots\ldots\ldots a_{n-1}\lambda + a_n$

Where $a_1, a_2, \ldots\ldots\ldots$ are real coefficients and then the Hurwitz matrix is

a. $H_1 = [a_1]$
b. $H_1 = [a_2]$
c. $H_1 = [a_3]$
d. None

17. For a linearised system of dynamical system if the eigen values λ_1 and λ_2 are real and equal then the system is asymptotically stable at origin if

a. $\lambda_1 = \lambda_2 < 0$
b. $\lambda_1 = \lambda_2 > 0$
c. Both A and B
d. None

18. For a non linear system of dynamical system if the eigen values λ_1 and λ_2 are real, unequal and both are negative then the equilibrium point is

a. Stable node
b. Sink
c. Attractor
d. All of the above

The linear system of equation $\dot{x} = 2x + y$, $\dot{y} = 3x + 4y$ have

a. Stable source
b. Unstable node
c. Saddle point
d. None of the above

For a non linear system of dynamical system if the eigen values λ_1 and λ_2 are complex having real part zero then the equilibrium point is

a. Center
b. Nutral center
c. Stable center
d. All of the above

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. Discuss the following models with equilibrium points, stability analysis constructing lyapunov function . also draw the graph and phase diagram **10**

$\dot{G} = -aG - bI + \alpha E + \delta$

$\dot{I} = cG - dI + \eta$

$\dot{E} = \beta E(1 - \gamma E)$

2. Consider the system **10**

$$\frac{dx}{dt} = x(1 - x + ky - k)$$
$$\frac{dy}{dt} = y(1 - y + ky - k)$$

where k is a constant different from 1 and -1.

The system above has exactly one equilibrium point (a,b) in the first quadrant with a>0 and b>0. Find this equilibrium point and Jacobian matrix

3. The interaction of two species of animals is modeled by **10**

$$dx/dt = x(2 - x + y)$$

$$dy/dt = y(4 - x - y)$$

for $x \geq 0$, $y \geq 0$

Sketch a phase portrait for this system. Make sure your sketch shows clearly the nullcines and the equilibria

4. State and prove Lyapunov stability theorem 10

5. Write a note on hyperbolic and non-hyperbolic equilibrium with different types of examples 10

6. There is one equilibrium point (a,b) with a>0 and b>0. Find the Jacobian matrix J of the system mentioned in Q.No2 at the point. 10

7. Elaborate saddle point and spiral with example showing graph. 10

8. Define with example: 10
   a. Stable
   b. Locally Stable
   c. Locally Asymptotically Stable
   d. Globally Stable

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