2022/12

SET A

M.Sc. MATHEMATICS
THIRD SEMESTER
MATHEMATICAL METHODS
MSM – 301

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. Full Marks: 70

(PART-A: Objective)

Time: 30 min. Marks: 20

Choose the correct answer from the following: *1x20=20*

1. The solution of integral equation $\int_0^x \frac{y(t)}{\sqrt{x-t}}\,dt = 1$ is

   a. $y(x) = \frac{1}{\pi\sqrt{x}}$
   b. $y(x) = \frac{1}{\sqrt{\pi}}$
   c. $y(x) = \frac{\pi}{\sqrt{x}}$
   d. $y(x) = \frac{\pi}{\sqrt{x^3}}$

2. The solution of integral equation $\int_0^x \frac{y(t)}{(x-t)}\,dt = \sqrt{x}$ is

   a. $y(x) = \frac{1}{2}$
   b. $y(x) = 1$
   c. $y(x) = \frac{3}{2}$
   d. $y(x) = \frac{3}{4}$

3. The function $k(x,t)$ is called as

   a. Constant of the integral equation
   b. Kernel of the integral equation
   c. Functional of the integral equation
   d. None

4. The values of λ for which determinant $\Delta = 0$ are known as

   a. Eigen values
   b. Eigen functions
   c. Kernels
   d. Separable kernels

5. The solution of Voltera integral equation $y(x) = 1 + \int_0^x y(t)\,dt$ is

   a. $y = e^x$
   b. $y = e^{-x}$

c. $y = \frac{e^x}{2!}$

d. $y = 1 + e^x$

6. The solutions corresponding to eigen values of λ can be expressed as
   a. Sum of eigen functions
   b. Difference of eigen functions
   c. Arbitrary multiples of eigen functions
   d. None

7. The resolvent kernel of integral equation with kernel $k(x,t) = 2x, (\lambda = 1)$ is
   a. $2e^{x^2-t^2}$
   b. $2xe^{x^2-t^2}$
   c. $2xe^{x-t}$
   d. $x-t$

8. The resolvent kernel of Voltera's integral equation $y(x) = f(x) + \lambda \int_0^x k(x,t)\, y(t)\, dt$ with kernel $k(x,t) = 1$ is
   a. $e^{\lambda(x-t)}$
   b. $e^{\lambda(x+t)}$
   c. -1
   d. 1

9. The integral equation $y(x) = 1 + \frac{1}{\pi} \int_0^{2\pi} \sin(x+t)\, y(t)\, dt$ has
   a. Infinite solution
   b. No solution
   c. Unique solution
   d. None

10. The value of α for which the integral equation $u(x) = \alpha \int_0^1 e^{x-t} u(t)\, dt$ has trivial solution
    a. -2
    b. -1
    c. 1
    d. 2

11. A Laplace Transform exists when
    a. Function is piece-wise continuous
    b. Function is of exponential order
    c. Both a and b
    d. None

12. The convolution process associated with the Laplace transform in time domain results
    a. Simple multiplication in complex frequency domain
    b. Simple division in complex frequency domain
    c. Simple multiplication in complex time domain
    d. Simple division in complex time domain

13. If $\frac{dy}{dx} + ay = e^{-bt}$ with initial condition *y(0)=0*. Then Laplace transform *y(s)* of the solution *y(t)* is

a. $\frac{1}{(s+a)(s+b)}$

b. $\frac{1}{b(s+a)}$

c. $\frac{1}{a(s+b)}$

d. None

14. The inverse Laplace transforms of $H(s)=\frac{s+3}{s^2+2s+1}$ *for* $t \geq 0$ is

a. $3te^{-t}+e^{-t}$

b. $3e^{-t}$

c. $2te^{-t}+e^{-t}$

d. $4te^{-t}+e^{-t}$

15. If the Laplace transform of function $f(t)$ is given by $\frac{s+3}{(s+1)(s+2)}$, then *f(0)* is

a. $\frac{3}{2}$

b. $\frac{1}{2}$

c. 0

d. 1

16. Fourier transform of $e^{-|x|}$ is $\frac{2}{1+p^2}$. Then what is the fourier transform of $e^{-2|x|}$

a. $\frac{4}{(4+p^2)}$

b. $\frac{2}{(4+p^2)}$

c. $\frac{2}{(2+p^2)}$

d. $\frac{4}{(2+p^2)}$

17. What is the fourier sine transform of e^{-ax} ?

a. $\frac{4}{4+p^2}$

b. $4\frac{a}{4a^2+p^2}$

c. $\frac{p}{a^2+p^2}$

d. $2\frac{p}{a^2+p^2}$

18. Find the fourier sine transform of $\frac{x}{a^2+x^2}$

a. $2\pi e^{-ap}$

b. $\frac{\pi}{2}e^{-ap}$

c. $\frac{2}{\pi}e^{-ap}$

d. πe^{-ap}

19. What is the fourier transform of $e^{ax}\left(a>0\right)$?

a. $\frac{p}{a^2+p^2}$

b. $2\frac{a}{a^2+p^2}$

c. $-2\frac{a}{a^2+p^2}$

d. Cannot be found

20. Laplace transform of t is

a. $\frac{1}{s}$

b. $\frac{1}{s^2}$

c. $\frac{1}{s^3}$

d. None

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(Descriptive)

Time : 2 hrs. 40 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Obtain the integral equation corresponding to the boundary value problem **10**

$$\frac{d^2\phi}{dx^2} + x\phi = 1; \phi(0) = 0, \phi(1) = 1$$

2. Solve the integral equation with the help of separable kernel **10**

$$\phi(x) = \lambda \int_{-1}^{1} \left(5xt^3 + 4x^2t + 3xt\right)\phi(t)\,dt$$

3. a. Find the Fourier series for the discontinuous function $f(x)$ defined by **3+2=5**

$$f(x) = \begin{cases} -k, -\pi < x < 0 \\ k, 0 < x < \pi \end{cases}$$

Hence show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$

b. Express the function $f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ as a Fourier integral **3+1=4**

and hence evaluate $\int_0^\infty \frac{\sin \lambda x \cos \lambda x}{\lambda} d\lambda$

c. State Fourier integral theorem. **1**

4. a. Using Convolution theorem, evaluate $L^{-1}\left\{\frac{1}{(s^2+4)(s+1)^2}\right\}$ 5+5=10

b. Solve the differential equation by Laplace Transform Technique

$$\frac{\partial y}{\partial t}=2\frac{\partial^2 y}{\partial x^2};y(0,t)=0,y(5,t)=0;y(x,0)=10\sin 4\pi x-5\sin 6\pi x$$

5. Find the Laplace transform of $2\frac{1}{2}\times 4=10$

(i) $t^2e^{-2t}cost$ (ii) $t^3e^{2t}sint$ (iii) $te^{2t}sin3t$
(iv) t^2e^{-2t}

6. Show that the resolvent kernel of the integral equation 10

$\phi(x)=1+\lambda\int_0^x xt\phi(t)\,dt$ is $xte^{\frac{\lambda}{3}(x^3-t^3)}$ and hence solve the equation.

7. Solve the integral equation 10

$$\phi(x)=1+x+\int_0^x (x-t)\phi(t)\,dt;\phi_0(x)=1$$

8. Solve the integral equation 10

$$\phi(x)=2x+\lambda\int_0^1 (x+t)\phi(t)\,dt;\phi_0(x)=1$$

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